4.1 Addition and resolution of forces

Let's begin Rock climbing

Climbers need to skilfully combine all the forces acting on their bodies to climb up rocks. Do you know how forces in different directions are combined?

1 Adding forces

When all the forces acting on an object are combined, the resultant force is the net force acting on the object.

When two or more forces act on an object, they can be combined and represented by a single force called the **resultant force**. The resultant force provides the same total effect of the original forces. Its magnitude can be larger or smaller than each of the original ones (Fig 4.1a).

(i) Forces applied in the same direction on the rope gives a stronger resultant force.

(ii) Forces applied in opposite directions on the stick gives a weaker resultant force

Fig 4.1a *Forces can be combined to give a stronger or a weaker resultant force*

a Parallel forces

In Chapter 3.1, we learned how to add two forces that are parallel. We first assign one direction as positive and the other as negative. The resultant force is then simply the sum of them (Fig 4.1b).

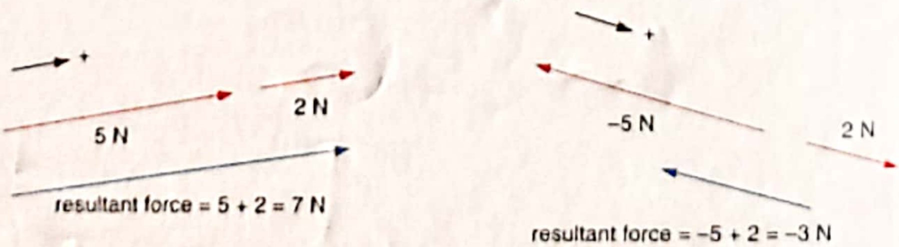


Fig 4.1b *Adding parallel forces*

resultant force 合力

b Forces in all directions

Simulation 4.1
Video 4.1

Like displacement, $\vec{F}$ denotes a vector while F represents the magnitude of a vector.

Forces are vectors and can be added graphically. To add two forces $\vec{F_1}$ and $\vec{F_2}$, we can draw a parallelogram using $\vec{F_1}$ and $\vec{F_2}$ as the adjacent sides (Fig 4.1c). The diagonal of the parallelogram, drawn from where the two forces act, represents the resultant force $\vec{F_1} + \vec{F_2}$. This is called the *parallelogram of forces method.*

We can also find the resultant force using the *'tip-to-tail' method*. To add two forces $\vec{F_1}$ and $\vec{F_2}$, we can shift $\vec{F_2}$ so that its tail is at the tip of $\vec{F_1}$. The resultant force is the vector from the tail of $\vec{F_1}$ to the tip of $\vec{F_2}$ (Fig 4.1d). This method is basically the same as the parallelogram of forces method.

In using the parallelogram of forces method, the tails of the forces ($\vec{F_1}$, $\vec{F_2}$ and $\vec{F_1} + \vec{F_2}$) meet at one point.

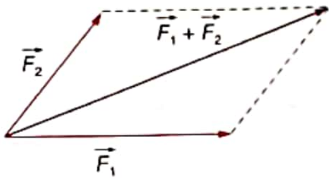


Fig 4.1c *Parallelogram of forces method.*

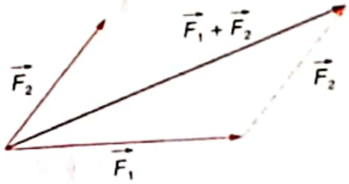
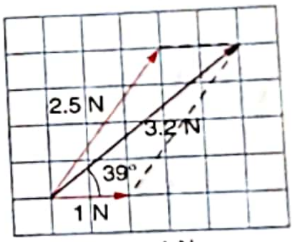


Fig 4.1d *Tip-to-tail method.*

Note that two forces give the largest resultant force when they act in the same direction; they give the smallest resultant force when they act in opposite directions.

If the forces are drawn in scale with the correct orientation, the resultant force can be drawn graphically (Fig 4.1e). Its magnitude and direction can be found by direct measurement.

In particular, if two forces are perpendicular to each other, the resultant force can be found by using Pythagoras' theorem and trigonometric ratios, as shown in Example 1.



scale: 1 cm = 1 N

Fig 4.1e *Finding the resultant force graphically.*

Simulation 4.2

Example 1 A stretched bow

A bow is stretched (Fig a). If the tension in the bowstring is 34 N and the two parts of the bowstring are perpendicular to each other, what is the resultant force acting on the arrow by the bowstring?

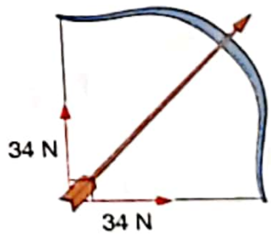


Fig a

Solution

Magnitude of resultant force = $\sqrt{34^2 + 34^2}$ = 48.1 N

$\tan\theta = \dfrac{34}{34} \Rightarrow \theta = 45°$

The resultant force is 48.1 N (inclined upwards by 45° to the horizontal).

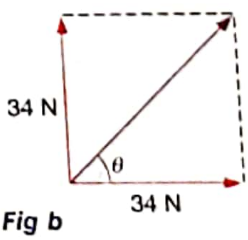


Fig b

Checkpoint 1 Q2 (p.152)

Example 2 Net force acting on a climber

Refer to the climber in **Let's begin** (Fig a). Figure b shows all the forces acting on him (represented by point C) to scale. Find graphically the net force acting on him.

Fig a

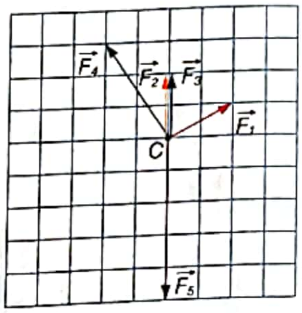
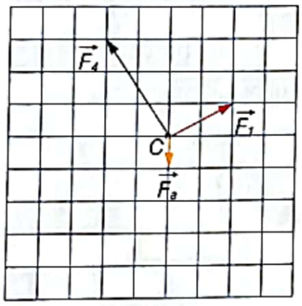
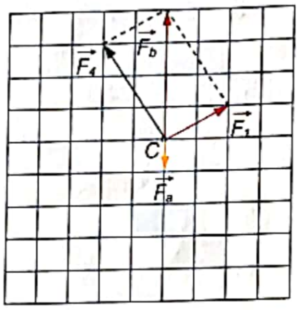


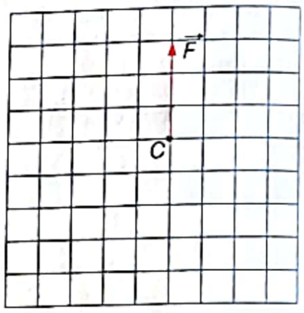
Fig b

Solution

The forces can be added in any order. Let us first add the parallel forces and then the non-parallel forces.







① Adding $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_5}$ gives $\vec{F_a}$.

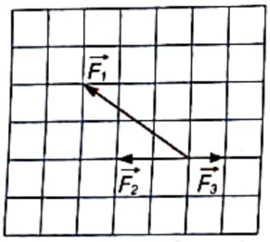
② Adding $\vec{F_1}$ and $\vec{F_4}$ gives $\vec{F_b}$.

③ Adding $\vec{F_a}$ and $\vec{F_b}$ gives net force $\vec{F}$.

Checkpoint 1 Q1 (p.152)

Checkpoint 1

1 Draw the resultant force in Figure a and find its magnitude.



scale: 1 cm = 1 N

Fig a

2 Two ropes are used to pull a tree as shown (Fig b). If the tension in each rope is 120 N, find the magnitude of the resultant force acting on the tree.

A 120 N
B 170 N
C 195 N
D 240 N

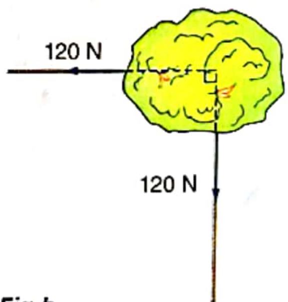


Fig b

Simulation 4.3

2 Resolving forces into components

When two forces are neither parallel nor perpendicular to each other, how can we find the resultant force algebraically? To answer this, we have to learn how to split a force first.

We can combine two forces into a single force. Conversely, we can split or **resolve** one force into two forces whose total effect is the same as that of the original one. The split forces are called the **components** of the original force. Usually we choose components that are perpendicular to each other.

Consider the force $\vec{F}$ in Figure 4.1f. To find its components along the x-direction and y-direction, draw a rectangle with $\vec{F}$ as the diagonal. The two sides of the rectangle represent the components $\vec{F_x}$ and $\vec{F_y}$.

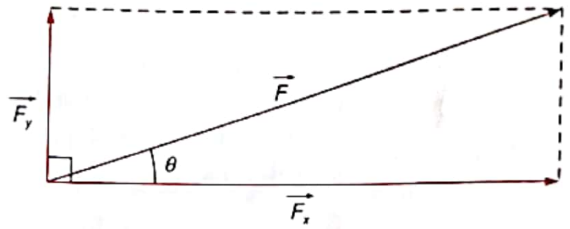


Fig 4.1f *Resolving a force into two perpendicular components.*

This method can be used only if the two components are perpendicular to each other.

▶ The magnitudes of the components in Figure 4.1f can be found by algebraic method:

$$\frac{F_x}{F} = \cos\theta \Rightarrow F_x = F\cos\theta$$

$$\frac{F_y}{F} = \sin\theta \Rightarrow F_y = F\sin\theta$$

Example 3 **Resolving a force algebraically**

A girl pulls her dog with a force of 10 N (Fig a). The string is 30° to the horizontal. What is the horizontal component of the pulling force?

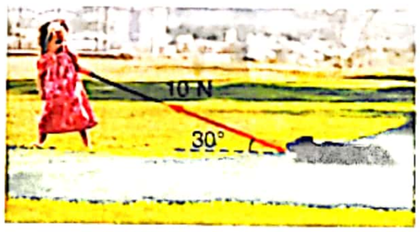


Fig a

Solution

Horizontal component of pulling force
$= F\cos 30°$
$= 10\cos 30°$
$= 8.66$ N

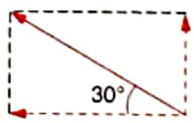


Fig b

Checkpoint 2 Q1 (p.154)

resolve 分解　component 分量

Skill

Angle of an inclined plane

Why is the angle β in Example 4 Figure b equal to 25°?

Consider a block on an inclined plane.

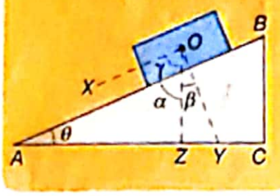
$OX \parallel AB \Rightarrow \gamma = \alpha$

$OX \perp OY \Rightarrow \beta = 90° - \gamma$

$OZ \perp AC \Rightarrow \theta = 90° - \alpha$

$= 90° - \gamma$

$= \beta$



A force can be resolved along directions other than the vertical and horizontal directions. The choice of directions usually depends on the moving direction of the object.

Example 4 Component of weight along a slope

A car of mass 1200 kg travels down a slope that is inclined at 25° to the horizontal (Fig a). The total resistive force acting on the car is 1000 N.

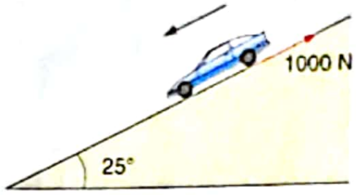


Fig a

(a) Find the component of the weight of the car along the slope.

(b) Find the resultant force acting on the car along the slope.

Solution

(a) The weight W of the car and its components parallel and perpendicular to the slope are shown in Figure b.

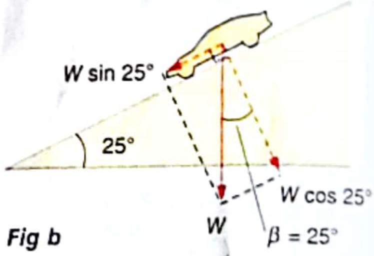


Fig b

Component of weight along slope

$= W \sin 25°$

$= 1200 \times 9.81 \times \sin 25°$

$= 4980$ N

(b) Take the direction down the slope as positive.

Resultant force $= 4980 - 1000$

$= 3980$ N

Checkpoint 2 Q2 (p.154)

Checkpoint 2

1 A woman pulls a suitcase with a force of 10 N as shown (Fig a). The angle between the force and the vertical is 60°. Find the horizontal component of the force.

Fig a

2 Consider a block of weight 15 N on an inclined plane (Fig b). Find the component of the weight of the block perpendicular to the plane.

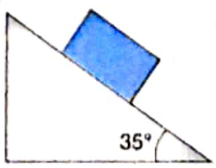


Fig b

3 Adding forces algebraically

When adding forces that are not perpendicular to each other, we cannot apply Pythagoras' theorem to find their resultant. However, we can find the resultant by first resolving the forces into their components.

Example 5 Net force on a bucket

A water bucket of weight 25 000 N is hung from a helicopter using a cable (Fig a). The cable makes an angle of 60° to the horizontal and the tension in the cable is 30 000 N. Neglect air resistance. Find the net force acting on the water bucket.

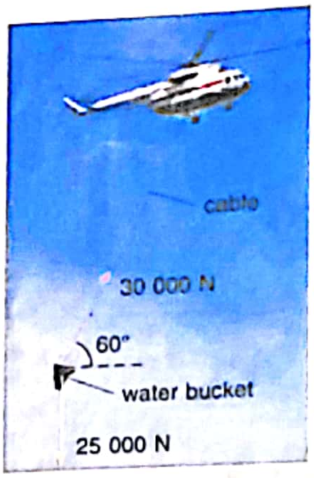


Fig a

Solution

Resolve the tension along the vertical and horizontal directions (Fig b).

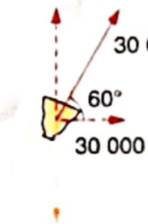


Fig b

In the vertical direction, taking upwards as positive,

resultant force $F_y = 30\ 000 \sin 60° - 25\ 000$
$= 981$ N

After a force F has been resolved into its components, F should be ignored in the rest of the calculation.

In the horizontal direction, taking the forward direction as positive,

resultant force $F_x = 30\ 000 \cos 60°$
$= 15\ 000$ N

Magnitude of net force F

$= \sqrt{F_x^2 + F_y^2}$

$= \sqrt{15\ 000^2 + 981^2}$

$= 15\ 030$ N $\approx 15\ 000$ N

$\tan\theta = \dfrac{F_y}{F_x} = \dfrac{981}{15\ 000}$

$\theta = 3.74°$

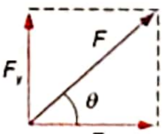


Fig c

The net force is 15 000 N inclined upwards at 3.74° to the horizontal.

Practice 4.1 Q6 (p.157)

We can do an experiment to verify this method of adding forces.

Video 4.2

Experiment 4a Addition of forces

1 Set up the apparatus as shown (Fig a). The rubber band is fixed to point *X*.

2 Pull the rubber band to the centre of the protractor *O* using the spring balances. Record the readings of the spring balances. Also record the angles *a* and *b*.

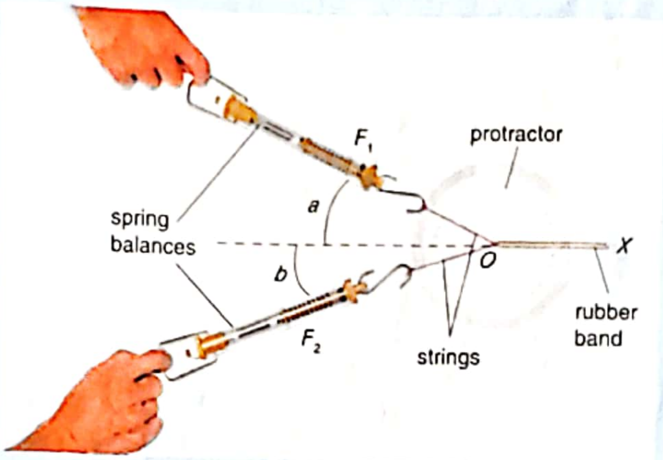


Fig a

3 Find the resultant of F_1 and F_2 using algebraic or graphical method.

4 Repeat by pulling the rubber band to *O* using different forces at different angles.

Why should the rubber band be stretched to the same position O in every trial?

Discussion

Is the resultant of F_1 and F_2 the same in every trial?

Checkpoint 3

1 Find the magnitude and direction of the resultant force acting on the particle (Fig a).

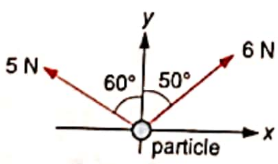


Fig a

2 Two 5-N forces act at the same point. What should the angle between the two forces be if the resultant is also 5 N in magnitude?

[Hint: The two forces and the resultant form an equilateral triangle.]

3 A block is sliding down a plane inclined at 30° to the horizontal (Fig b). The weight of the block is 20 N and the friction acting on it is 8 N. Find the net force acting on it along the plane.

[Hint: What is the direction of the friction?]

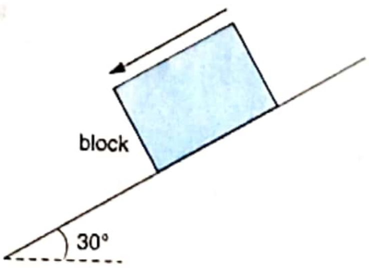
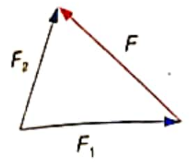


Fig b

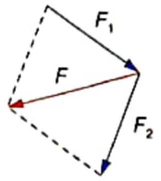
Practice 4.1

1 Which of the following diagrams correctly show(s) the resultant force F of two forces F_1 and F_2?

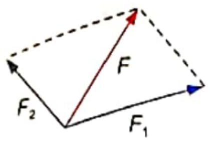
(1)



(2)



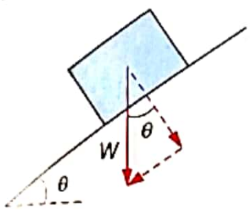
(3)



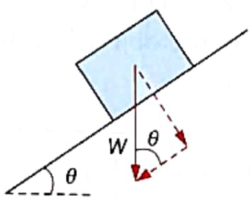
A (2) only B (3) only
C (1) and (2) only D (2) and (3) only

2 An object of weight W is placed on an inclined plane. Which of the following diagrams correctly shows how W is resolved into components parallel and perpendicular to the inclined plane (components shown as dotted lines)?

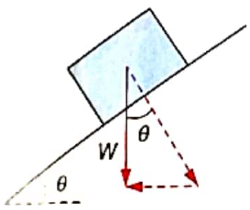
A



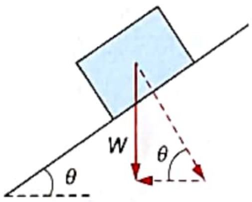
B



C

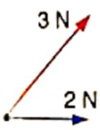


D

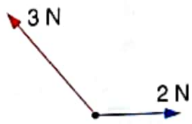


★3 A 2-N force and a 3-N force act on the same point in different ways as shown. According to the magnitudes of the resultant forces, arrange the four cases in ascending order.

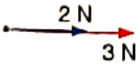
(1)



(2)



(3)



(4)

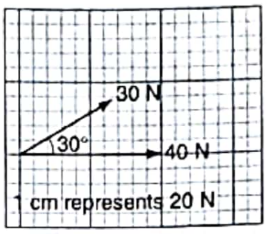
A (1), (3), (2), (4) B (1), (4), (2), (3)
C (4), (1), (2), (3) D (4), (2), (1), (3)

★4 The magnitude of the resultant force of a 3-N force and a 4-N force cannot be

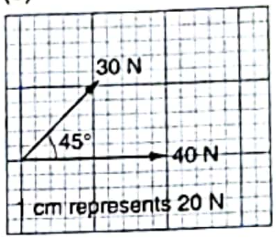
A 0 N. B 1 N.
C 4 N. D 7 N.

5 For each pair of forces, find their resultant graphically.

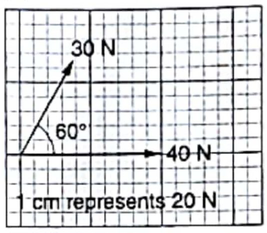
(a)



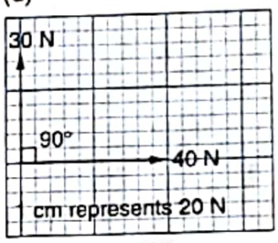
(b)



(c)



(d)



6 Complete Q5 using the algebraic method.

7 Force $\vec{F}$ is the resultant of $\vec{F_1}$ and $\vec{F_2}$. Figure a shows $\vec{F_1}$ and $\vec{F}$. Draw $\vec{F_2}$ on Figure a.

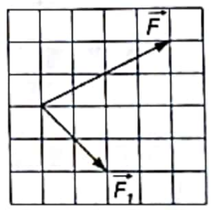


Fig a

★8 An object is acted on by three forces. The resultant of the three forces is zero. The figure below shows two of the forces (Fig b). Draw the third force in the figure.

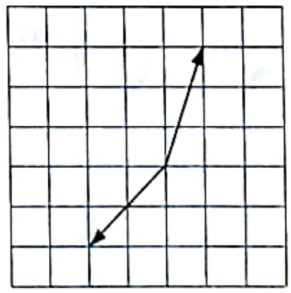


Fig b

4.2 Forces in a plane and Newton's laws of motion

Let's begin **Helicopters**

Have you ever seen a helicopter accelerate forwards? It inclines forwards instead of staying horizontal. Do you know why?

A video showing that a helicopter accelerates from rest:
http://www.youtube.com/watch?v=LiuRP_9CJZ4

In Chapter 3, we learned how to apply Newton's laws of motion to simple cases in which the forces acting on an object are parallel. Now we shall study the more general case in which the forces are coplanar (i.e. act in the same plane).

When an object remains at rest or moves uniformly, by Newton's first law of motion, the resultant of the coplanar forces acting on it must be zero, i.e. there is no net force acting on the object in **any direction**.

Example 6 **T-shirt hung on a rope**

A T-shirt is hung on a rope (Fig a). The total weight of the T-shirt and the hanger is 5 N. Find the tension T in the rope.

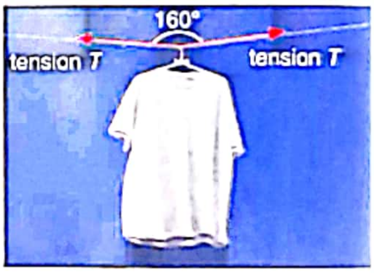


Fig a

Solution

Figure b shows the free-body diagram for the T-shirt and the hanger.

Since they are stationary, the net force acting on them along the vertical direction is zero.

$$T \cos 80° + T \cos 80° - 5 = 0$$

$$T = 14.4 \text{ N}$$

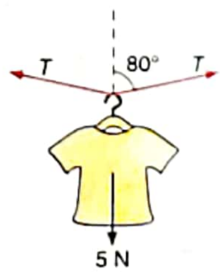


Fig b

Checkpoint 4 Q1 (p.160)

Example 7 Lifting a box with a pulley

A student of mass 50 kg holds a box of mass 20 kg with a light rope and a smooth pulley (Fig a). The student and the box remain at rest.

(a) Draw the free-body diagrams for the student and the box.

(b) Find the tension in the rope.

(c) Find the normal reaction acting on the student by the ground.

(d) Find the magnitude of the force that the student exerts on the ground.

(e) Can the student stand still if the ground is smooth? Explain briefly.

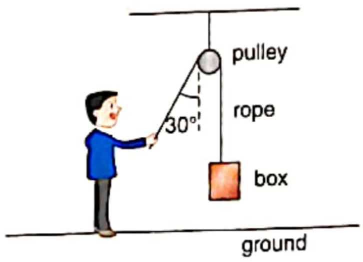


Fig a

Solution

Take the upward direction and the direction to the right as positive.

(a)

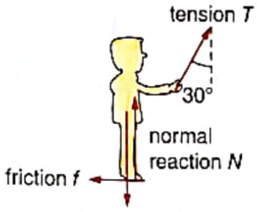


Fig b

Fig c

(b) Since the box is at rest, the net force acting on it is zero.

$T - W_B = 0$

$\Rightarrow\ T = W_B = 20 \times 9.81 = 196.2 \text{ N} \approx 196 \text{ N}$

The tension in the rope is 196 N.

(c) Since the student is at rest, the net force acting on him is zero.

Consider the vertical direction.

$N + T\cos 30° - W_S = 0$

$\Rightarrow\ N = W_S - T\cos 30° = 50 \times 9.81 - 196.2\cos 30° = 320.6 \text{ N} \approx 321 \text{ N}$

The normal reaction by the ground is 321 N.

(d) The net force acting on the students is zero in the horizontal direction.

$T\sin 30° - f = 0$

$\Rightarrow\ f = T\sin 30° = 196.2 \sin 30° = 98.1 \text{ N}$

Resultant force acting on him by the ground

$= \sqrt{N^2 + f^2} = \sqrt{320.6^2 + 98.1^2} = 335 \text{ N}$

By Newton's third law, he exerts a force of 335 N on the ground.

(e) If the ground is smooth, the horizontal component of the tension will be the net force acting on the student. By Newton's second law, he will accelerate towards the right and cannot stand still.

Practice 4.2 Q7 (p.167)

Example 8 Finding a force graphically

A board is hung by 2 strings and remains stationary as shown (Fig a). Figure b shows the weight $\vec{W}$ of the board and the tension $\vec{T_1}$ in scale.

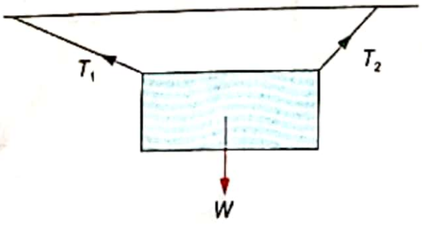


Fig a

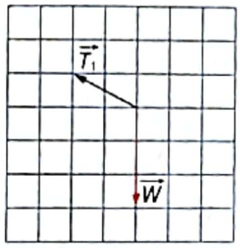


Fig b

(a) Find the resultant force of $\vec{T_1}$ and $\vec{W}$ graphically.

(b) Draw tension $\vec{T_2}$ on the graph in (a).

Solution

(a) The resultant force of $\vec{T_1}$ and $\vec{W}$ can be found by the parallelogram of forces method (Fig c).

(b) Since the board is stationary, the net force acting on it is zero. That means $\vec{T_2}$ balances the force $\vec{T_1} + \vec{W}$. Therefore, $\vec{T_2}$ has the same magnitude but acts in the opposite direction to $\vec{T_1} + \vec{W}$ (Fig c).

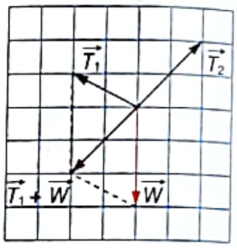
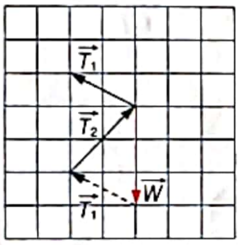


Fig c

Checkpoint 4 Q2 (p.160)

Alternative method:
The three forces form a triangle when they are added using the tip-to-tail method. This helps us draw $\vec{T_2}$ easily. ▶



Checkpoint 4

1 A block of mass 2 kg moves down steadily along a rough plane inclined at 30° to the horizontal (Fig a). Find the normal reaction acting on the block by the plane.

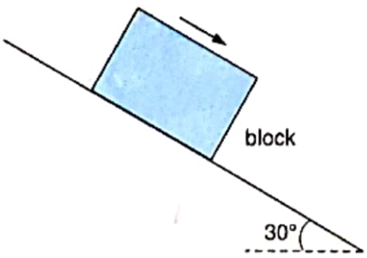


Fig a

2 An object is held stationary by four forces as shown (Fig b).

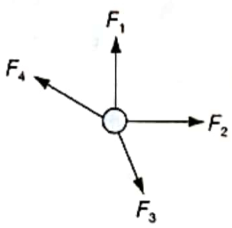


Fig b

The force F_4 disappears suddenly and the three other forces remain unchanged. Draw on the figure the direction of the net force acting on the object at the instant that F_4 disappears.

When an object experiences coplanar forces that are not balanced, according to Newton's second law of motion, it accelerates along the direction of the net force. We may study the motion of the object by applying Newton's second law after finding the net force.

Sometimes, we are only interested in the motion along a particular direction. In this case, the coplanar forces are resolved into components to find the net force in that direction. This net force determines the acceleration of the object in that direction.

Simulation 4.4

Example 9 Acceleration of a sledge

A girl sitting on a sledge is pulled by a boy and accelerates horizontally (Fig a). The string is at an angle of 40° to the horizontal. The tension in the string is 55 N. The friction on the sledge is 30 N. The total mass of the girl and the sledge is 50 kg. Find the acceleration of the sledge.

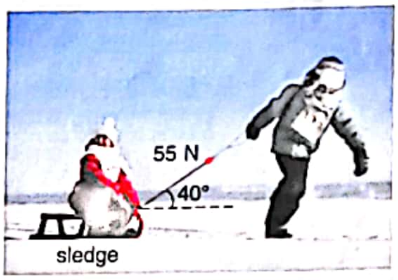


Fig a

Solution

The free-body diagram for the girl on the sledge is shown in Figure b. Resolve the tension along the vertical and horizontal directions. Since the sledge accelerates horizontally, we only need to consider the horizontal direction.

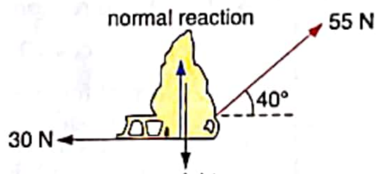


Fig b

Take the direction towards the right as positive.

Net force along the horizontal direction = 55 cos 40° − 30 = 12.13 N

By $F = ma$,

$$a = \frac{F}{m} = \frac{12.13}{50} = 0.243 \text{ m s}^{-2}$$

The acceleration of the sledge is 0.243 m s^{-2} towards the right.

Practice 4.2 Q3 (p.166)

Skill

Solving problems of coplanar forces with Newton's 2nd law

① Draw the free-body diagram for the object of interest.

② To resolve the forces, choose two directions that are perpendicular and convenient to solve the problem, e.g. the object's moving direction.

③ Define the positive directions.

④ Find the net force in each direction.

⑤ Apply the equation $F = ma$ and solve for the unknowns.

(Steps ③ to ⑤ are similar to Steps ② to ⑤ in Skill on p.111 in Chapter 3.)

Example 10 Pushing a trolley up a slope

A man pushes a trolley up a slope (Fig a). He applies a force of 200 N parallel to the slope. The slope is at an angle of 10° to the horizontal. The total mass of the trolley including its contents is 50 kg. The friction acting on the trolley by the slope is 80 N.

(a) Find the magnitude of the normal force acting on the trolley by the slope.

(b) Find the acceleration of the trolley.

Fig a

Solution

(a) Figure b shows the free-body diagram for the trolley.

Resolve the weight W along directions parallel and perpendicular to the slope (Fig c).

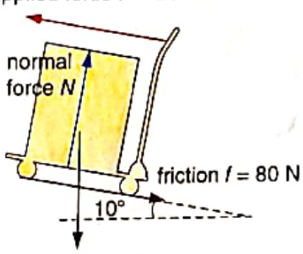


Fig b

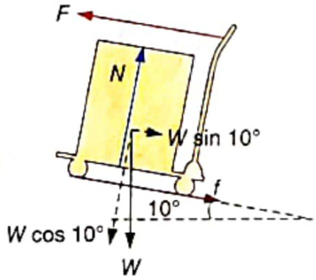


Fig c

> N and f in the free-body diagram are the resultants of the corresponding forces acting on the wheels.
>
> Note that we do not draw the components of a force in a free-body diagram.

The object does not move along the direction perpendicular to the slope. This means that the net force in this direction is zero.

Take the direction of the normal force N as positive.

$$N - W\cos 10° = 0$$

$\Rightarrow$ Normal force = $W\cos 10° = 50 \times 9.81 \times \cos 10° = 483$ N

(b) Consider the direction along the slope.

Take the direction up the slope as positive.

Net force along the slope $= F - W\sin 10° - f$
$= 200 - 50 \times 9.81 \times \sin 10° - 80$
$= 34.83$ N

By $F = ma$,

$$a = \frac{F}{m} = \frac{34.83}{50} = 0.697 \text{ m s}^{-2}$$

The acceleration of the trolley is 0.697 m s^{-2} up the slope.

Practice 4.2 Q11 (p.167)

Example 11 Acceleration of a helicopter

The uplifting force always acts perpendicularly to the plane of the blades.

An uplifting force U acts on a helicopter (Fig a). The total mass of the helicopter and its load is 5400 kg. The helicopter travels along a straight line horizontally. Assume air resistance can be neglected.

Fig a

(a) What is the magnitude of the uplifting force U?

(b) Find the net force acting on the helicopter.

(c) What is the acceleration of the helicopter?

(d) Suppose the uplifting force becomes 6×10^4 N and the angle between this force and the vertical remains 20°. What is the magnitude of the acceleration of the helicopter?

Solution

Figure b shows the free-body diagram for the helicopter. Take the upward direction and the direction towards the left as positive.

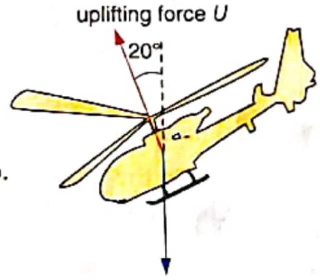


Fig b

(a) Since the helicopter does not move vertically, the net force along the vertical direction is zero.

$$U \cos 20° - mg = 0$$

$$\Rightarrow U = \frac{mg}{\cos 20°} = \frac{5400 \times 9.81}{\cos 20°} = 5.64 \times 10^4 \text{ N}$$

(b) $F_x = U \sin 20°$

$= 5.64 \times 10^4 \times \sin 20°$

$= 1.93 \times 10^4$ N

The net force is 1.93×10^4 N towards the left.

Refer to the question in **Let's begin**. A helicopter inclines forwards when it accelerates because it needs the horizontal component of the uplifting force to provide the net force for acceleration.

(c) By $F = ma$,

$$a = \frac{F}{m} = \frac{1.93 \times 10^4}{5400} = 3.57 \text{ m s}^{-2}$$

The acceleration of the helicopter is 3.57 m s^{-2} towards the left.

(d) Net force along vertical direction F_y

$= U \cos 20° - mg = 6 \times 10^4 \times \cos 20° - 5400 \times 9.81 = 3410$ N

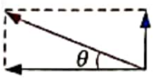
Net force along horizontal direction F_x

$= U \sin 20° = 6 \times 10^4 \times \sin 20° = 2.05 \times 10^4$ N

Magnitude of net force

$= \sqrt{F_x^2 + F_y^2} = \sqrt{(2.05 \times 10^4)^2 + 3410^2} = 2.08 \times 10^4$ N

Magnitude of acceleration $= \frac{F}{m} = \frac{2.08 \times 10^4}{5400} = 3.85$ m s^{-2}



$\tan \theta = \frac{3410}{2.05 \times 10^4}$

$\theta = 9.44°$

The helicopter accelerates along a direction 9.44° above the horizontal.

Practice 4.2 Q9 (p.167)

Experiment skill

Example 12 Friction on trolley along an inclined plane

A student does an experiment to investigate the friction acting on a trolley of mass 0.5 kg when it moves on an inclined plane (Fig a). A motion sensor is installed at the upper end of the plane to record the motion of the trolley.

The student gives the trolley a push. The trolley moves up the plane and then slides back down. Figure b shows the v–t graph of the trolley.

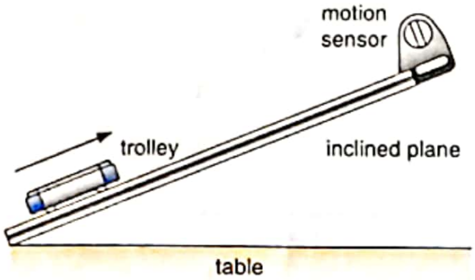


Fig a

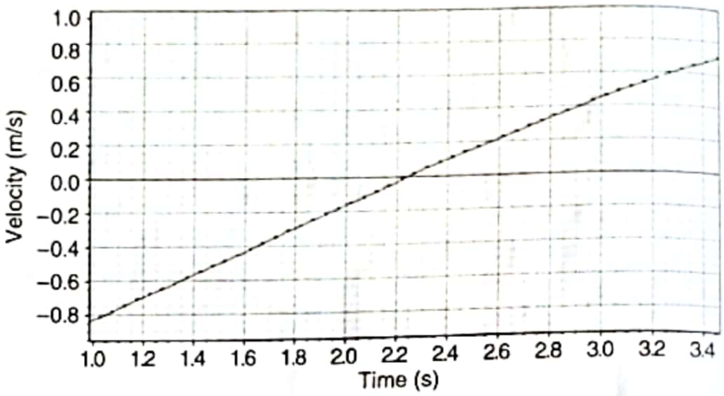


Fig b

(a) State a precaution in this experiment.

(b) Estimate the accelerations of the trolley during the upward journey and the downward journey.

(c) Estimate the friction acting on the trolley during the upward journey. The plane is inclined at 3.6° to the horizontal.

Solution

Take the direction down the plane as positive.

(a) Do not push the trolley too hard to avoid the trolley hitting the motion sensor. / The trolley should move along a straight line towards and away from the motion sensor.

(b) Acceleration during upward journey a_U

$= \text{slope of graph before 2.25 s} = \dfrac{-0.10 - (-0.60)}{2.10 - 1.35} = 0.667 \text{ m s}^{-2}$

◀ During the upward journey, the trolley moves towards the motion sensor, so its velocity is negative. That means the trolley is moving upwards before 2.25 s.

Acceleration during downward journey a_D

$= \text{slope of graph after 2.25 s} = \dfrac{0.50 - 0.10}{3.10 - 2.40} = 0.571 \text{ m s}^{-2}$

(c) Figure c shows the forces acting on the trolley during the upward journey. Consider the direction along the plane. By $F = ma$,

$W \sin\theta + f = ma_U$

$f = ma_U - W\sin\theta$

$= 0.5 \times 0.667 - 0.5 \times 9.81 \times \sin 3.6°$

$= 0.0255$ N

The friction is 0.0255 N.

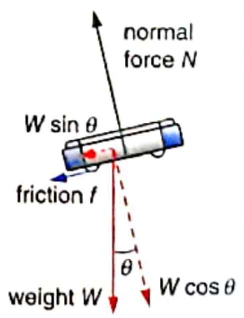
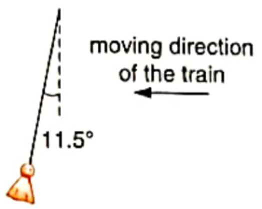


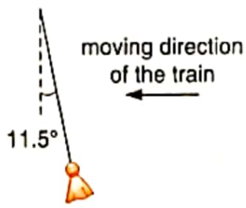
Fig c

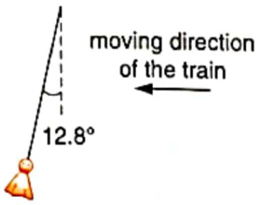
Revision exercise Q27 (p.174)

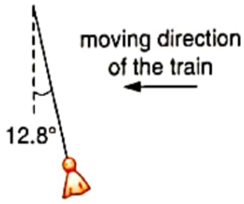
Exam link 1 Acceleration of a hanging ornament

Alan holds a hanging ornament inside an MTR train. He finds that when the train accelerates, the cord hanging the ornament is at an angle to the vertical. If the train moving towards the left slows down at 2 m s^{-2}, which of the following graphs best represents the position of the ornament?

A


B


C


D


Solution

For options A and C, the forces acting on the ornament are shown in Figure a. For options B and D, the forces acting on the ornament are shown in Figure b.

The net horizontal force ($T \sin \theta$) points towards the right in Figure a while the net horizontal force points towards the left in Figure b.

Since the train slows down towards the left, its acceleration points towards the right.

∴ The net force acting on the ornament also points towards the right.

∴ Options B and D are incorrect.

Since the ornament does not moves along the vertical direction, the net force along the vertical direction is zero.

$$T \cos \theta - mg = 0$$

$$T \cos \theta = mg \quad \text{.............. (1)}$$

Consider the horizontal direction. Take the direction towards the right as positive. By $F = ma$,

$$T \sin \theta = ma \quad \text{....................... (2)}$$

(2) ÷ (1),

$$\tan \theta = \frac{a}{g} = \frac{2}{9.81}$$

$$\Rightarrow \theta = 11.5°$$

∴ The answer is A.

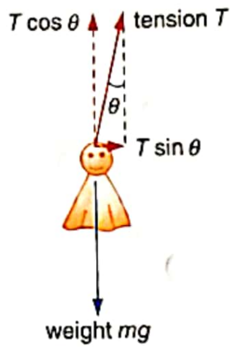


Fig a

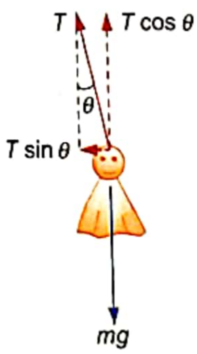


Fig b

Note that the angle of the cord is independent of the mass of the ornament.

Revision exercise Q12 (p.170)

Checkpoint 5

1 A man pushes a box on the ground with a force of 50 N (Fig a). His force acts at an angle of 60° to the horizontal. The mass of the box is 25 kg. The friction between the box and the ground is 20 N. What is the acceleration of the box?

Fig a

2 A box slides down a rough plane which is inclined to the horizontal at 30° (Fig b). The acceleration of the box is 0.5 m s^{-2}. The mass of the box is 2 kg. Find the friction between the box and the plane.

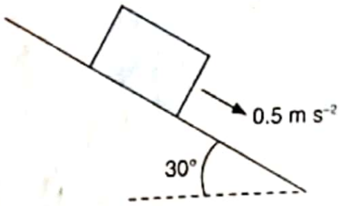


Fig b

Practice 4.2

If necessary, take g = 9.81 m s^{-2}. Unless otherwise specified, assume air resistance to be negligible.

(For Q1–2.) A 2-kg block is given a sharp push such that it moves up a rough inclined plane (Fig a). The block stops after moving for a certain distance. The friction between the block and the plane is 10 N when the block is moving up the plane.

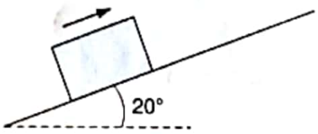
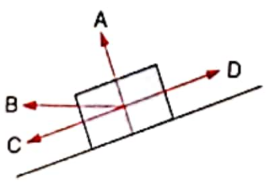


Fig a

1 What is the direction of the net force acting on the block when it is moving up the plane?



2 What is the friction acting on the block when it is at rest on the plane?

A 6.71 N
B 7.14 N
C 10 N
D 18.4 N

★ 3 Some steel rods are hung by two ropes as shown (Fig b). The tension in each rope is 600 N and the weight of the steel rods is 1000 N. What is the acceleration of the steel rods?

A 0.385 m s^{-2} (upwards)
B 1.96 m s^{-2} (upwards)
C 0.0392 m s^{-2} (downwards)
D 3.92 m s^{-2} (downwards)

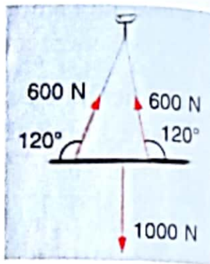


Fig b

★ 4 Three forces, F_1, F_2 and F_3, act on an object. Figure c shows F_1, F_2 and the direction of acceleration a of the object.

Which of the following graphs best shows F_3?

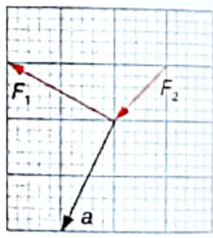
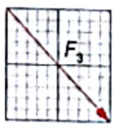
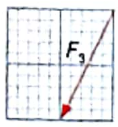
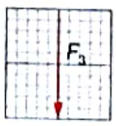
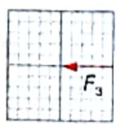


Fig c

A


B


C


D


★ 5 If three forces act on a body and the body remains stationary, which of the following must be true?

(1) The three forces are parallel to each other.
(2) The three forces have the same magnitude.
(3) The resultant of any two forces is in the opposite direction to the third one.

A (1) only B (3) only
C (1) and (3) only D (2) and (3) only

★ 6 To remove a car from a ditch, the driver ties a rope tightly between the car and a tree. The driver then pulls at the centre of the rope with a force of 500 N and the car begins to move at the instant shown in Figure d. Find the tension in the rope.

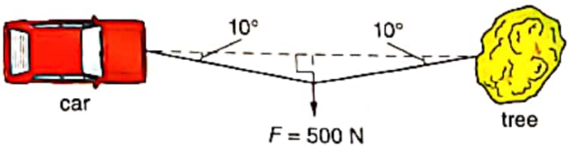


Fig d

★ 7 A dog tied by a string to a railing is trying to get away as shown (Fig e). It is at rest at the moment shown.

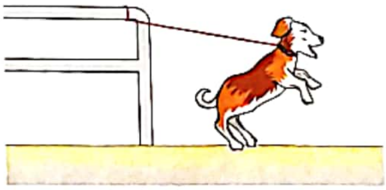


Fig e

(a) Draw the free-body diagram for the dog.
(b) Explain briefly the direction of the friction acting on the dog by the ground.

★ 8 Some dandelion seeds are blown into the sky (Fig f). At a certain instant, the blowing force acting on one of the seeds is 6×10^{-6} N at an angle of 35° to the vertical. The mass of the seed is 0.4 mg.

Fig f

(a) What is the magnitude and direction of the net force acting on the seed?
(b) What is the acceleration of the seed?

★ 9 The V-22 Osprey (Fig g) is an aircraft that can take off and land vertically. This is because the orientation of the rotors can be changed. Suppose the total mass of the aircraft with load is 20 000 kg.

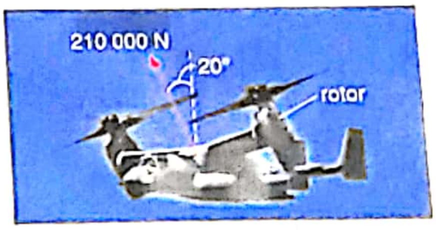


Fig g

(a) During landing, the rotors points vertically upwards. If the aircraft slows down at 1 m s^{-2} when it is moving vertically downwards, what is the force produced by the rotors?
(b) The aircraft takes off again. At a certain instant, the rotors are tilted to produce an upward force of 210 000 N that acts at an angle of 20° to the vertical (Fig g). Find the acceleration of the aircraft.

★ 10 A girl holding a kite on a line runs forwards (Fig h). The kite line is at an angle of 20° to the horizontal. The tension in the kite line is 5.8 N. The force acting on the kite by air is 6.2 N and is at 30° above the horizontal. The mass of the kite is 100 g. The forces all lie in the same plane. Find the acceleration of the kite.

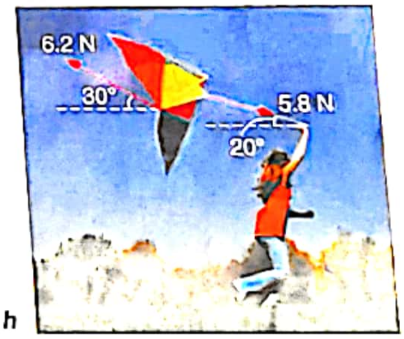


Fig h

★ 11 A 2-kg trolley runs down an inclined runway with a constant speed after being given a slight push.

(a) If the runway is at an angle of 10° to the horizontal, what is the frictional force acting on the trolley?
(b) Are the component of the weight of the trolley along the runway and the friction acting on the runway by the trolley an action and-reaction pair? Explain briefly.
(c) If the runway is now set at 30°, what will the acceleration of the trolley be as it moves down the runway? Assume the frictional force remains the same as that in (a).

Review 4

Terms

1 component 分量 p.153
2 resolve 分解 p.153
3 resultant force 合力 p.150

Main points

4.1 Addition and resolution of forces

1 Two or more forces can be added to find the resultant force using the parallelogram of forces method (Fig a) or the tip-to-tail method (Fig b):

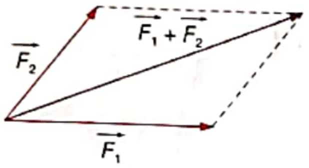


Fig a

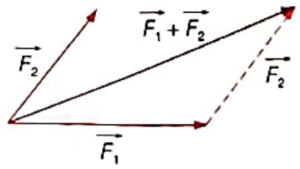


Fig b

2 A force can be resolved into two components (Fig c).

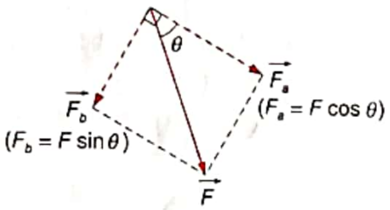


Fig c

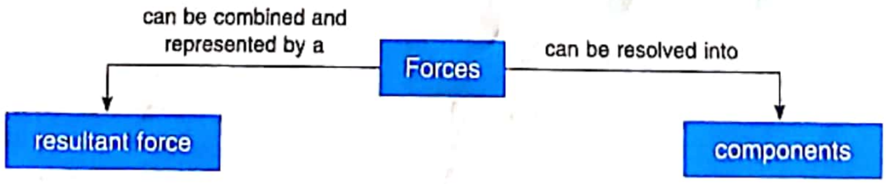
3 Forces in arbitrary directions can be added by first resolving them into their components.

4.2 Forces in a plane and Newton's laws of motion

4 When an object remains at rest or moves uniformly, there is no net force acting on it in any direction.

5 When coplanar forces act on an object, the acceleration of the object can be found by first finding out the net force and then applying Newton's second law.

Concept map



Revision exercise 4

If necessary, take $g = 9.81$ m s^{-2}. Unless otherwise specified, assume air resistance to be negligible.

Concept traps

(For Q1–2.) Determine whether each of the following statements is true or false.

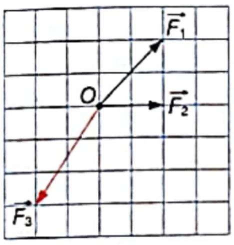
1 When a force is resolved, the components must be perpendicular to each other.

2 When a force F is resolved, the magnitude of each component must be smaller than that of F.

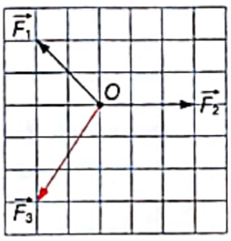
Multiple-choice questions

3 In which of the following cases is the net force acting on object O zero?

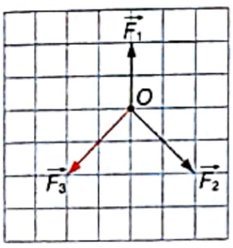
A



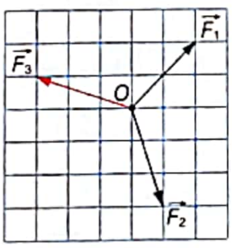
B



C



D



★ 4 A force of 20 N acts perpendicularly on a block placed on a rough inclined plane (Fig a). The mass of the block is 3 kg. What is the normal reaction acting on the block by the plane?

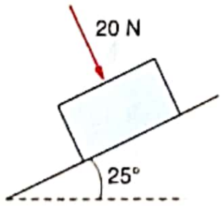


Fig a

A 6.67 N B 20 N
C 32.4 N D 46.7 N

★ 5 A trolley travels down an inclined plane from rest. Its v–t graph is shown in Figure b.

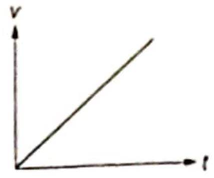
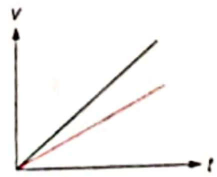


Fig b

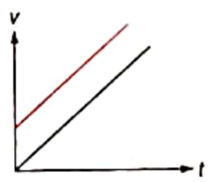
Then the inclined angle of the plane is decreased and the trolley travels down the plane from rest again. Which of the following graphs (in red line) best shows the new v–t graph of the trolley?

A

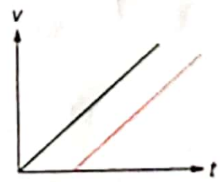
B



C



D



★ 6 Two blocks of masses M and m are connected by an inextensible string and at rest on the two smooth surfaces of wedge ABC as shown (Fig c). AB is 50 cm long and BC is 30 cm long. The string passes over a smooth pulley. Find the ratio of $M : m$.

(This video may give you some hints: http://www.youtube.com/watch?v=nDKGHGdXLEg)

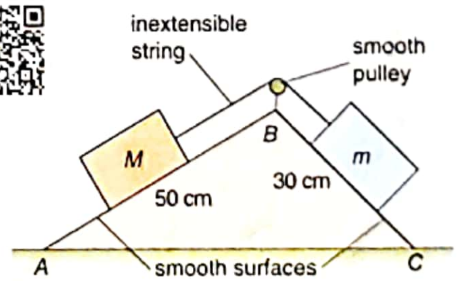


Fig c

A 9 : 25
B 3 : 5
C 5 : 3
D 25 : 9

★ 7 Three masses are hung over two pulleys as shown below (Fig d). If W is at rest, find the tension T in the string holding W.

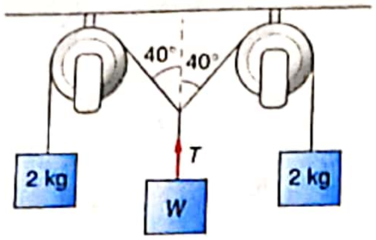


Fig d

A 25.2 N
B 30.1 N
C 32.9 N
D 39.2 N

★ 8 A student lets block X slide down an inclined plane. The friction acting on X is 0.2 times its weight. Then she uses another block Y with a larger mass and the friction acting on it is also 0.2 times its weight when it slides down the plane. Which of the following statements about the two blocks when they slide down the plane is/are correct?

(1) If the blocks accelerate, the net force acting on Y will be larger than that on X.
(2) If the blocks accelerate, the acceleration of Y will be larger than that of X.
(3) If X slides down the plane at a constant velocity, Y will also slide down at a constant velocity.

A (1) only
B (1) and (3) only
C (2) and (3) only
D (1), (2) and (3)

★★ 9 A girl of mass 20 kg sits in a car that accelerates constantly at 1.5 m s^{-2} up an inclined road (Fig e). The inclination of the road is 10°. What is the magnitude of the force acting on the girl by the car?

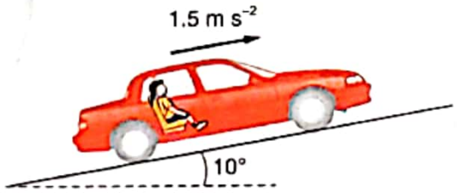
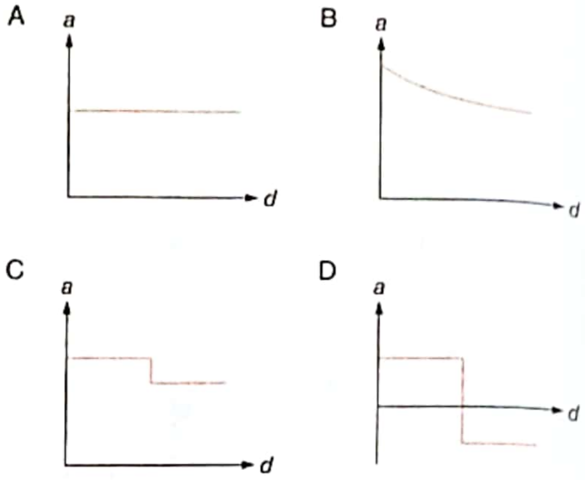


Fig e

A 30.0 N
B 64.1 N
C 193 N
D 204 N

Refer Eg 11 (p.163)

★★ 10 A trolley is given a sharp push so that it moves up a rough inclined plane. It rolls back down the plane after reaching its highest position. Which of the following graphs best illustrates how the acceleration a of the trolley varies with the distance travelled d?



Refer Eg 12 (p.164)

11 **HKALE 2005 Paper 2 Q1**

A block of mass 2.5 kg rests on the rough surface of a wedge, which in turn rests on a rough horizontal floor as shown. What is the frictional force exerted by the floor on the wedge if the whole system is at rest?

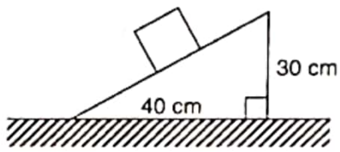


Fig f

A 0 N
B 12 N
C 15 N
D 20 N

12 **HKCEE 2010 Paper 2 Q31**

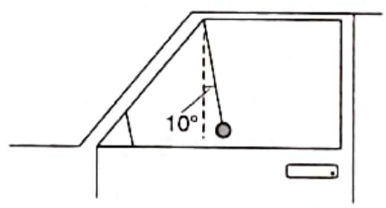


Fig g

In the figure above, a ball is hanging inside a car by a string. When the car accelerates on a horizontal road, the string makes an angle of 10° with the vertical. Find the magnitude of the acceleration of the car.

A 1.74 m s^{-2}
B 1.76 m s^{-2}
C 5.67 m s^{-2}
D 9.85 m s^{-2}

13 **HKDSE 2012 Paper 1A Q10**

A block of mass *m* resting on a 30° incline is given a slight push and slides down the incline with a uniform speed. Which of the following statements about the block's motion on the incline is/are correct?

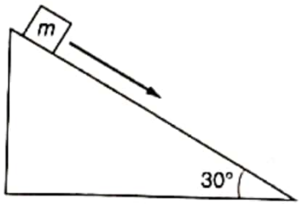


Fig h

(1) There is no net force acting on the block.
(2) The frictional force acting on the block is 0.5*mg*.
(3) If the block is given a greater initial speed, it will slide down the incline with acceleration.

A (1) only
B (3) only
C (1) and (2) only
D (2) and (3) only

14 **HKDSE 2013 Paper 1A Q5**

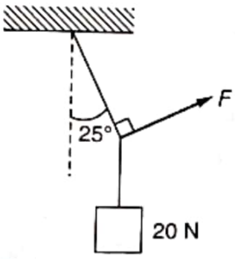


Fig i

A block of weight 20 N is suspended by a light string from the ceiling. A force *F* is applied such that the block is displaced to one side with the string making an angle of 25° with the vertical as shown. Find the magnitude of *F*.

A 8.5 N
B 9.3 N
C 18.1 N
D 47.3 N

15 **HKDSE 2014 Paper 1A Q4**

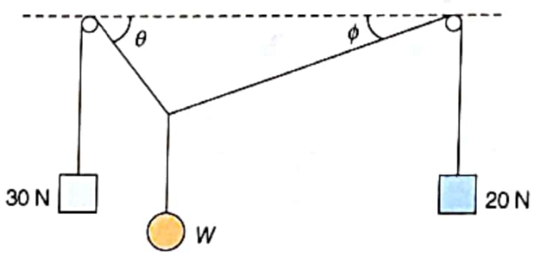


Fig j

The figure shows a weight *W* attached to two light strings which pass over two smooth pegs at the same height and with weights 30 N and 20 N attached to the respective ends of each string. The system is at equilibrium. Which deduction about *W* is correct?

A *W* is less than 50 N.
B *W* is equal to 50 N.
C *W* is greater than 50 N.
D No information about *W* can be obtained as angles θ and ϕ are not known.

Conventional questions

16 Several forces act on a block as shown below (Fig k).

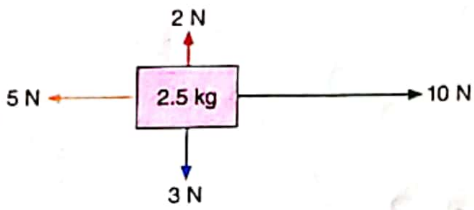


Fig k

(a) What is the net force acting on the block in
(i) the vertical direction, (1 mark)
(ii) the horizontal direction? (1 mark)

(b) What is the net force acting on the block? (2 marks)

(c) What is the acceleration of the block? (2 marks)

17 A girl is sliding down a slide (Fig l). Her mass is 45 kg. The slide is inclined at 50° to the horizontal.

Fig l

(a) Draw the free-body diagram for the girl. (2 marks)

(b) What is the direction of the net force acting on her if she
(i) comes down the slide faster and faster? (1 mark)
(ii) comes down slowly at a constant speed? (1 mark)
(iii) sits on the slide and does not move? (1 mark)

(c) What is her acceleration if the friction between her and the slide is 300 N? (2 marks)

✶ 18 A mass is supported by two spring balances as shown in Figure m. The mass is stationary. The weight of the mass is 10 N.

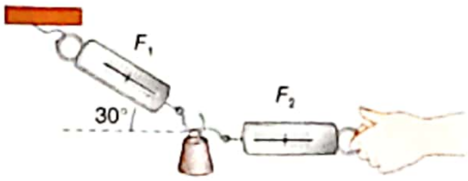


Fig m

(a) Draw the free-body diagram for the mass. (2 marks)

(b) What is the net force acting on the mass? (1 mark)

(c) Find the readings F_1 and F_2 of the balances. (3 marks)

✶ 19 A man is standing on a cable (Fig n). The tension in the cable is 1280 N.

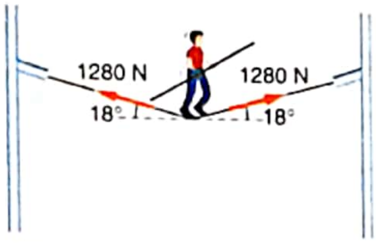


Fig n

(a) Sketch a vector diagram of the tensions acting on the man and their resultant force. (2 marks)

(b) Calculate the magnitude of the resultant force of the tensions. (2 marks)

(c) What is the weight of the man? (1 mark)

✶ 20 Four horizontal forces act on an object placed on a smooth horizontal surface as shown (Fig o). The object remains stationary. The mass of the object is 20 kg.

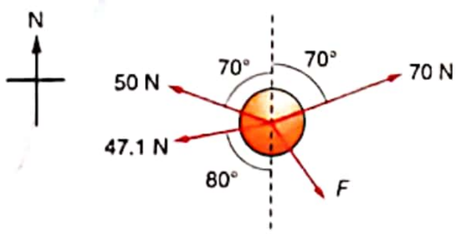


Fig o

(a) Find the magnitude of F. (3 marks)

(b) Find the direction of F. (2 marks)

(c) If the 70-N force disappears suddenly, what is the magnitude and direction of the acceleration of the object at that instant? Assume that the three other forces remain unchanged. (3 marks)

✶ 21 A trolley of mass 1 kg is released from rest on a rough inclined runway (Fig p). The runway is at an angle of 20° to the horizontal. The trolley accelerates uniformly as it moves down the runway.

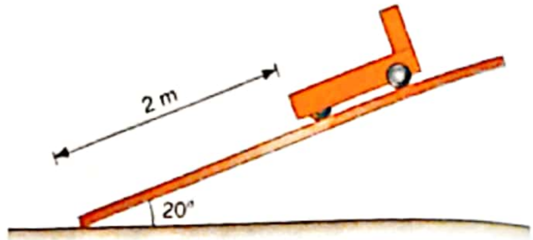


Fig p

(a) (i) Draw the free-body diagram for the trolley. (2 marks)

(ii) The trolley takes 4 s to travel 2 m to the bottom of the runway. What is its acceleration? (2 marks)

(iii) Find the friction acting on the trolley. (2 marks)

(b) Now the runway is adjusted so that the trolley moves down the runway at a uniform speed. A student thinks that if the trolley is given a sharp push up the runway, it will continue to move up at a uniform speed. Explain whether the student is correct. (3 marks)

✶ 22 A trolley of mass 0.4 kg is projected up a rough inclined plane with an initial speed of 4 m s^{-1} at time t = 0 (Fig q). It reaches its highest position at t = 1.6 s before sliding back down the plane. The plane is at an angle of 10° to the horizontal.

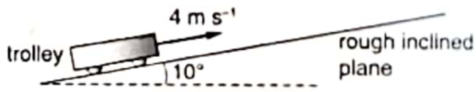


Fig q

(a) Find the acceleration of the trolley as it moves up the plane. (2 marks)

(b) Find the friction acting on the trolley as it moves up the plane. (2 marks)

(c) Assume that the friction acting on the trolley has the same magnitude during the upward and downward journeys.

(i) Find the speed of the trolley when it returns to its starting position. (4 marks)

(ii) Sketch the v–t graph of the trolley from t = 0 to the instant it returns to its starting position. (4 marks)

★ **23** The figure below shows a crane lifting an object (Fig r). Assume that the mass of the object is 225 kg. Each of the four ropes connected to the object are at an angle of 40° to the vertical and have the same tension.

Fig r

(a) The object is held stationary.

(i) What is the net force acting on the object? (1 mark)

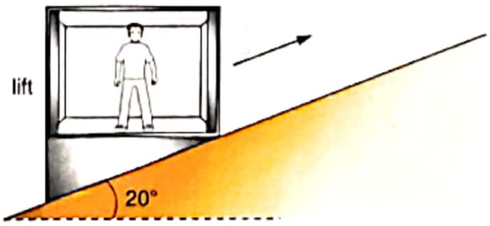
(ii) What is the tension in each rope? (2 marks)

(b) If the object is raised upwards with an acceleration of 0.4 m s^{-2}, what is the tension in each rope? (2 marks)

(c) A rope may break if the tension in it is too large. Is it safer to hang the object with longer or shorter ropes? Explain briefly. (3 marks)

★★ **24** An engineer designs a lift that takes passengers up a slope along a straight line (Fig s). The floor of the lift is kept horizontal.

Fig s



Suppose a man of mass 65 kg takes the lift up the slope. Find the normal force *N* and friction *f* (magnitude and direction) acting on the man by the floor of the lift when the lift

(a) accelerates at 0.5 m s^{-2}; (4 marks)

(b) moves at a constant velocity of 1.2 m s^{-1}; (2 marks)

(c) decelerates at 0.8 m s^{-2}. (2 marks)

[Hint: Acceleration can be resolved into components.]

Refer p.161

★★ **25** Block *X* is placed on a smooth inclined plane while block *Y* is placed on block *X* (Fig t). The mass of *X* is 2 kg and the mass of *Y* is 0.5 kg. The smooth plane is at an angle of 15° to the horizontal.

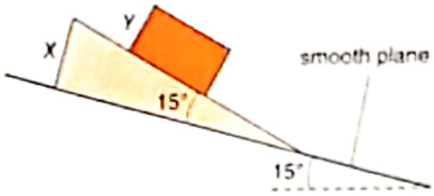


Fig t

(a) At first, *X* is held stationary and *Y* is at rest on *X* because of the friction between them.

(i) Draw the free-body diagram for *Y*. (2 marks)

(ii) Find the magnitude of the friction acting on *Y*. (2 marks)

(b) Then *X* is released. *Y* moves together with *X* without slipping.

(i) What is the magnitude and direction of the acceleration of *Y*? (3 marks)

(ii) What is the magnitude and direction of the force acting on *X* by *Y*? (5 marks)

Refer Eg 11 (p.163)

26 **HKCEE 2008 Paper 1 Q9**

Figure u shows a cable car system for transporting passengers from station *A* to station *B* on the top of a hill.

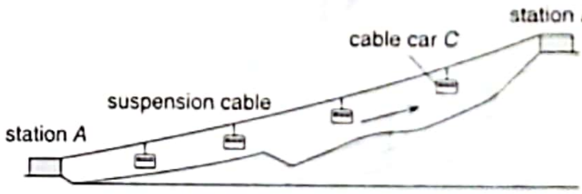


Fig u

(a) The mass of the cable car *C* is 600 kg. State the magnitude and the direction of its weight. (2 marks)

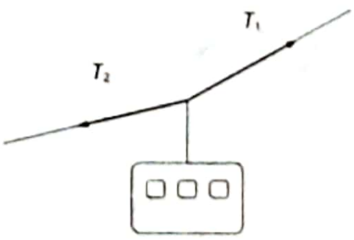


Fig v

Self test 4

Time allowed: 15 minutes   Total: 9 marks

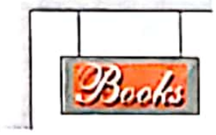
Instructions

1 Answer ALL questions.
2 Section A consists of multiple-choice questions. Section B contains a conventional question.
3 Write your answers in the space provided.
4 For data, formulae and relationships, refer to Appendix.

Section A

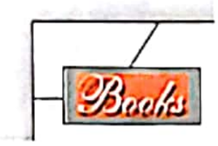
1 A signboard is hung by two identical wires. Which of the following is the safest way to hang it?

A

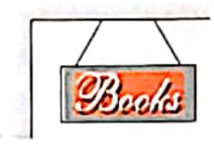


B

C



D



☐

2 Blocks X and Y are placed on a smooth plane inclined at 30°. A horizontal force of 50 N acts on X (Fig a). The weight of X is 20 N and the weight of Y is 30 N. Find the magnitude of the force acting on X by Y.

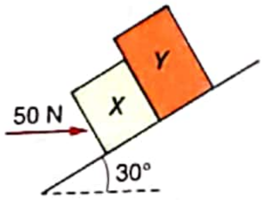


Fig a

A 0   B 7.32 N
C 26.0 N   D 36.6 N

☐

Section B

3 A 800-g box is connected to a 500-g mass by an inextensible string running over a smooth pulley (Fig b). When the mass is released, the box is pulled towards the left. The friction between the box and the table is 3.5 N. The box stops before reaching the edge of the table.

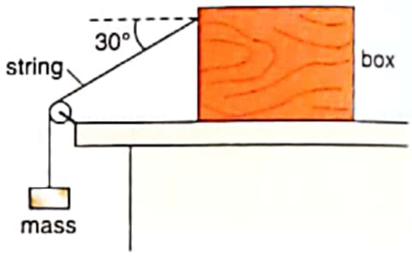


Fig b

(a) At the instant shown, what are the tension in the string and the acceleration of the box? (4 marks)

___

___

___

(b) Find the angle between the string connected to the box and the horizontal at the instant that the box changes from accelerating to decelerating motion. (3 marks)

___

___

___